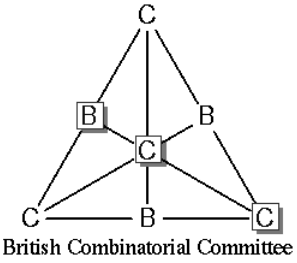
The Open University



British Combinatorial Committee

Open University Winter Combinatorics Meeting

Wednesday 2 February 2011

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Winter Combinatorics Meeting

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The talks will take place in Christodoulou Meeting Room 15, on the Open University campus in Milton Keynes.

Timetable

10:15 - 10:45	Tea/Coffee (in the Mathematics and Statistics Common Room)
10:50 - 10:55	Welcome and introduction – Robin Wilson
11:00 - 11:40	Sophie Huczynska (University of St Andrews) *From sum-free sets to subgroups: what lies between?*
11:45 - 12:25	Mark Jerrum (Queen Mary, University of London) *The computational complexity of Ising model on some classes of matroids*
12:30 - 13:55	Lunch
14:00 - 14:40	Deryk Osthus (University of Birmingham) *Perfect matchings and Hamiltonian cycles in hypergraphs*
14:45 - 15.25	Tom McCourt (University of Queensland, moving to University of Bristol) *Biembedding twofold triple systems*
15:30 - 15:55	Tea/Coffee (in the Mathematics and Statistics Common Room)
16:00 - 16:40	Colin McDiarmid (University of Oxford) *Colouring random geometric graphs*

The meeting is financially supported by the British Combinatorial Committee.

Drawing of St Michael's church, on the OU campus, by Jini Williams.

Abstracts

From sum-free sets to subgroups: what lies between?

Sophie Huczynska (University of St Andrews)

Given a finite abelian group G and a subset S of G, the size $r(S)$ of the set $\{(a, b) : a, b, a + b \in S\}$ may range between 0 and $|S|^2$, with the extremal values corresponding to sum-free subsets and subgroups of G. In this talk, I will discuss the spectrum of intermediate values which $r(S)$ may take, particularly in the setting where G is $\mathbb{Z}/p\mathbb{Z}$ under addition (p prime).

The computational complexity of Ising model on some classes of matroids

Mark Jerrum (Queen Mary, University of London)

Classically, the Ising model in statistical physics is defined on a graph. But through the random cluster formulation we can make sense of the Ising partition function in the wider context of an arbitrary matroid. I expect most of the talk will be spent setting the scene. But eventually I'll come round to discussing the computational complexity of evaluating the partition function on various classes of matroids (graphic, regular and binary). I'm not a physicist nor a card-carrying matroid theorist, so the talk should be reasonably accessible. This is joint work with Leslie Goldberg (Liverpool).

Perfect matchings and Hamiltonian cycles in hypergraphs

Deryk Osthus (University of Birmingham)

Since there is (probably) no hypergraph analogue of Tutte's theorem on perfect matchings, it makes sense to search for natural sufficient conditions which guarantee a perfect matching. Together with Kühn and Treglown, I recently determined the minimum vertex degree that ensures a perfect matching in a 3-uniform hypergraph. This solves a problem of Han, Person and Schacht.

In my talk, I will outline the proof of this result and introduce related open problems. In particular, I will also discuss conditions which guarantee a Hamilton cycle in a hypergraph.

Biembedding twofold triple systems

Tom McCourt (University of Queensland, moving to University of Bristol)

Constructions due to Ringel show that there exists an orientable face 2-colourable triangular embedding of the complete graph on n vertices (equivalently an orientable biembedding of two Steiner triple systems of order n) for all $n \equiv 3 \,(\mathrm{mod}\ 12)$. Similarly, constructions due to Youngs show that there exists an orientable biembedding of two Steiner triple systems of order n for all $n \equiv 7 \,(\mathrm{mod}\ 12)$.

In this presentation I will discuss results establishing the existence of an orientable face 2-colourable triangular embedding of the twofold complete graph on n vertices (equivalently an orientable biembedding of a pair of twofold triple systems of order n) for all $n \equiv 4 \,(\mathrm{mod}\ 12)$.

Colouring random geometric graphs

Colin McDiarmid (University of Oxford)

From independent random points $X_1, \ldots, X_n$ in $\mathbb{R}^d$, drawn according to some probability distribution ν on $\mathbb{R}^d$, and a positive distance $r = r(n) > 0$, we may construct a random geometric graph G_n with vertex set $\{1, \ldots, n\}$ where distinct i and j are adjacent when $\|X_i - X_j\| \leq r$. Here $\|.\|$ may be an arbitrary norm on $\mathbb{R}^d$ and we allow any probability distribution ν with a bounded density function.

Recall that a proper colouring of a graph G is a colouring of the vertices such that adjacent vertices receive different colours, and the chromatic number $\chi(G)$ is the least number of colours in a proper colouring; and a clique is a set of vertices which are pairwise adjacent, and the clique number $\omega(G)$ is the largest cardinality of a clique (note that $\omega(G)$ is a trivial lower bound for $\chi(G)$).

For the random geometric graph G_n, we consider the clique number $\omega(G_n)$, the chromatic number $\chi(G_n)$ and the ratio of these two quantities, all for large n. We find that there are constants $\omega(t)$ and $\chi(t)$ such that $\frac{\omega(G_n)}{\sigma n r^d} \to \omega(t)$ and $\frac{\chi(G_n)}{\sigma n r^d} \to \chi(t)$ almost surely when $\sigma n r^d \sim t \ln n$. Here σ denotes the maximum density of the probability distribution ν. Thus in this case the ratio $\frac{\chi(G_n)}{\omega(G_n)} \to \chi(t)/\omega(t)$ almost surely (and we may allow any $t \in [0, \infty]$). A striking feature is that (except for less interesting choices of the norm when the unit ball tiles $\mathbb{R}^d$) there is a "threshold": there is a constant t_0 with $0 < t_0 < \infty$ such that $\chi(t)/\omega(t) = 1$ for $t \leq t_0$ and $\chi(t)/\omega(t) > 1$ for $t > t_0$.

This is joint work with Tobias Müller (and builds on earlier work of Mathew Penrose and the speaker).