Open University
Winter Combinatorics Meeting

Wednesday 30 January 2008

The talks will take place in room CMR11 on the Open University campus in Milton Keynes.

Timetable

10:15 - 10:45	Tea/Coffee (in M Block, Room Q229)
10:50 - 10:55	Welcome and introduction Chris Earl, Dean of the Faculty of Mathematics, Computing and Technology
11:00 - 11:40	Martin Mačaj, Comenius University, Bratislava *2-perfect trail systems*
11:45 - 12:25	Gordon Royle, University of Western Australia *Colourings (and other homomorphisms) of cubelike graphs*
12:30 - 13:55	Lunch
14:00 - 14:40	James Hirschfeld, University of Sussex *The size of the automorphism group of an algebraic curve*
14:45 - 15.25	Andrew Thomason, University of Cambridge *Turan's theorem, Szemeredi's lemma and colours*
15:30 - 15:55	Tea/Coffee (in M Block, Room Q229)
16:00 - 16:40	Anthony Hilton, Queen Mary, London *k-to-1 continuous functions between graphs*

The meeting is financially supported by the British Combinatorial Committee.

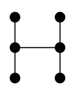
Abstracts

k-to-1 continuous functions between graphs

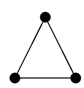
Anthony Hilton, Queen Mary, London

It is well-known that a simple graph can be represented quite nicely in $\mathbb{R}^3$ with the edges being straight-line segments. A k-to-1 function $f : G \to H$ from a graph G onto a graph H has the property that $|f^{-1}(p)| = k$ for each point p in H. Here the points can be vertices, or they can be points on the edges. Given G, H and an integer k, is there a k-to-1 continuous function (= map) from G onto H ? Failing that, is there a finitely discontinuous k-to-1 function from G onto H? The answers to these questions often have a heavily combinatorial flavour. We shall describe some of what is known about this subject, and some current research.

By way of getting started, can you find a 2-to-1 continuous function from any of $(0, 1), (0, 1], [0, 1]$ onto any of $(0, 1), (0, 1], [0, 1]$? What about a 3-to-1 continuous function? What caught my attention to this topic was the question (unsolved at the time), of whether there is a 3-to-1 continuous function from



onto



The size of the automorphism group of an algebraic curve

James Hirschfeld, University of Sussex

If $\mathcal{F}$ is an algebraic curve defined over a field K, then a K-automorphism is an invertible polynomial map fixing $\mathcal{F}$; the set of these automorphisms is the *automorphism group* G of $\mathcal{F}$. For example, when $\mathcal{F}$ is rational, that is, a line in the plane, then $G \cong \mathrm{PGL}(2, K)$.

A classical bound, that is, for curves over the complex numbers when the genus $g \geq 2$, is that

$$|G| \leq 84(g - 1).$$

This bound is achieved in some cases.

When $K = \mathbf{F}_q$, what can be said about $|G|$? Results on this will be surveyed.

2-perfect trail systems

Martin Mačaj, Comenius University, Bratislava

Two aspects of 2-perfect trail systems will be discussed:

(1) 2-perfect trail systems and total colourings.

(2) Constructing 4-cycle systems from 2-perfect trail systems.

In the first part we show that there is a correspondence between 2-perfect trail systems and total colourings of complete graphs. In the second part we present a construction of 4-cycle systems on n^2 vertices from a 2-perfect trail system on n vertices, such that resulting 4-cycle systems have very large number of trades.

Colourings (and other homomorphisms) of cubelike graphs

Gordon Royle, University of Western Australia

A cube-like graph is defined to be a Cayley graph of the elementary abelian group Z_2^n. In this talk, I will discuss the colouring properties of cube-like graphs, show how the theory of graph homomorphisms can be used to establish some of these properties with remarkably little effort and outline some open problems on both colourings and homomorphisms of cube-like graphs.

Turan's theorem, Szemeredi's lemma and colours

Andrew Thomason, University of Cambridge

This talk (about joint work with Ed Marchant) concerns a version of extremal graph theory in which edges have colours. The coloured version contains the classical theory; the extension to coloured graphs has various motivations but the principal application is to the use of Szemeredi's Lemma. It can also be seen as an extension of classical extremal graph theory to induced subgraphs (whereas the classical theory is concerned with forbidden subgraphs, not necessarily induced).

Let H be a fixed graph whose edges are coloured red and blue. Let G be a large graph, each of whose edges can be red, blue or both. We associate a weight to each edge of G; red edges have weight p, blue edges weight q, and edges of both colour have weight $p + q$. We normalize so that $p + q = 1$. The extremal question is this: how large must the total edge weight of G be in order to guarantee that G contains a copy of H?

We discuss some small special cases, in particular, only when H and G are complete graphs. In this case, the answer is known when H has at most 4 vertices (due to Richer) or when the red edges of G form a star (Richer, and also Diwan-Mubayi and Balogh-Martin). We discuss the case when the blue graph of H is $K_{3,3}$. A solution to this case for $\frac{1}{3} \leq q \leq \frac{2}{3}$ implies a solution to the edit-distance problem of Alon-Stav (a solution also given by Balogh-Martin). We give a simple solution for $\frac{1}{3} \leq q$; the problem becomes harder for small q, but we describe a general method, and show how to attain $\frac{1}{8} \leq q$.