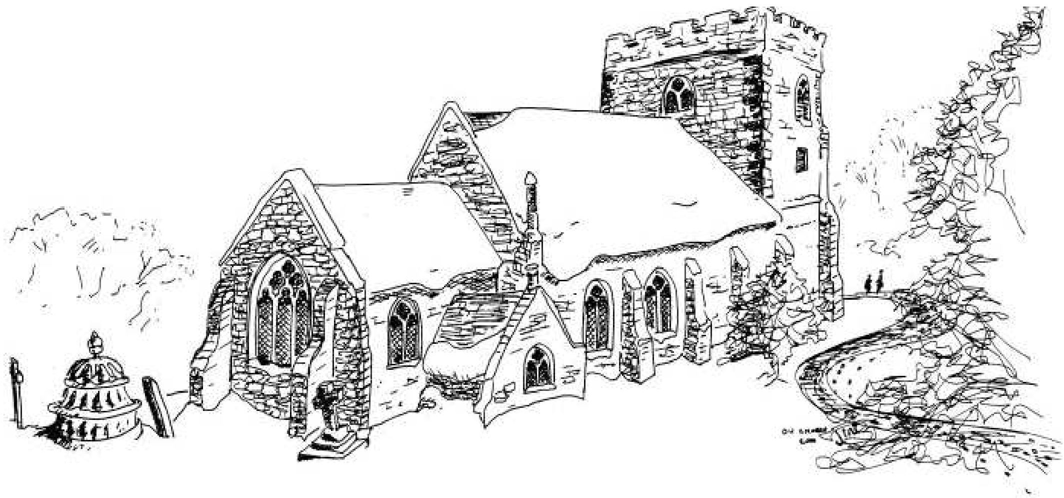
Open University
Winter Combinatorics Meeting

Wednesday 29 January 2014



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The talks will take place in Christodoulou Meeting Room 15, on the Open University campus in Milton Keynes.

Timetable

10:15 - 11:00	Tea/Coffee (in the Mathematics and Statistics Common Room)
11:00 - 11:40	Daniel Kral (University of Warwick) *Graph limits and finite forcibility*
11:45 - 12:25	Alex Scott (University of Oxford) *Hypergraphs of bounded disjointness*
12:30 - 13:55	Lunch
14:00 - 14:40	Pinar Heggernes (University of Bergen, Norway) *Enumeration in graph classes*
14:45 - 15.25	Brett Stevens (Carleton University, Canada) *Optimising an imperfect tournament*
15:30 - 15:55	Tea/Coffee (in the Mathematics and Statistics Common Room)
16:00 - 16:40	Misha Rudnev (University of Bristol) *The sum-product problem*

The meeting is financially supported by the London Mathematical Society and the British Combinatorial Committee.

Drawing of St Michael's church, on the OU campus, by Jini Williams.

Abstracts

Graph limits and finite forcibility

Daniel Kral (University of Warwick)

Limits of discrete structure keep attracting a substantial attention inside the discrete mathematics community. In the talk, we will present basic notions related to the case of limits of dense graphs (those with quadratic number of edges) as introduced in a series of papers by Borgs, Chayes, Lovasz, Sos, Szegedy and Vesztergombi. We will then focus on various aspects of finite forcibility of such limits, i.e., when such a limit is determined by densities of finitely many subgraphs. We will present counterexamples to two conjectures of Lovasz and Szegedy related to finitely forcible graphs. At the end of the talk, we will discuss limits of graphs that need not be dense.

Hypergraphs of bounded disjointness

Alex Scott (University of Oxford)

A k-uniform hypergraph is said to be intersecting if no pair of edges is disjoint. The maximal size of an intersecting k-uniform hypergraph with a given groundset is given by the beautiful and well-known theorem of Erdos, Ko and Rado.

A k-uniform hypergraph is s-almost intersecting if every edge is disjoint from exactly s other edges. Gerbner, Lemons, Palmer, Patks and Szcsi made a conjecture on the maximal number of edges in such a hypergraph. We prove a strengthened version of this conjecture and determine the extremal graphs. We also give some related results and conjectures.

Joint work with Elizabeth Wilmer.

Enumeration in graph classes

Pinar Heggernes (University of Bergen, Norway)

Enumerating, counting, and determining the maximum number of various objects in graphs have long been established as important areas within graph theory and graph algorithms. As the number of enumerated objects is very often exponential in the size of the input graph, enumeration algorithms fall into two categories depending on their running time: those whose running time is measured in the size of the input, and those whose running time is measured in the size of the output. Based on this, we concentrate on the following two types of algorithms.

1. Exact exponential time algorithms. The design of these algorithms is mainly based on recursive branching. The running time is a function of the size of the input graph, and very often it also gives an upper bound on the number of enumerated objects any graph can have.

2. Output polynomial algorithms. The running time of these algorithms is polynomial in the number of the enumerated objects that the input graph actually contains. Some of these algorithms have even better running times in form of incremental polynomial or polynomial delay, depending on the time the algorithm spends between each consecutive object that is output.

The methods for designing the two types of algorithms are usually quite different. Common to both approaches is that efforts have traditionally mainly been concentrated on arbitrary graphs, whereas graphs with particular structure have largely been left unattended. In this talk we look at enumeration of objects in graphs with special structure. In particular, we focus on enumerating minimal dominating sets in various graph classes.

Algorithms of type 1: The number of minimal dominating sets that any graph on n vertices can have is known to be at most $1.7159n$. This upper bound might not be tight, since no examples of graphs with $1.5705n$ or more minimal dominating sets are known. For several classes of graphs, like chordal, split, and interval graphs, we improve and tighten the upper bound on the number of minimal dominating sets. At the same time, we give algorithms for enumerating all minimal dominating sets, where the running time of each algorithm is within a polynomial factor of the proved upper bound for the graph class in question. In some cases, we provide examples of graphs containing the maximum possible number of minimal dominating sets for graphs in that class, thereby showing the corresponding upper bounds to be tight.

Algorithms of type 2: Enumeration of minimal dominating sets in graphs has very recently been shown to be equivalent to enumeration of minimal transversals in hypergraphs. The question whether the minimal transversals of a hypergraph can be enumerated in output polynomial time is a fundamental and challenging question; it has been open for several decades and has triggered extensive research. We show that all minimal dominating sets of a line graph can be generated in incremental polynomial, and consequently output polynomial, time. We are able to improve the delay further on line graphs of bipartite graphs. Finally we show that our method is also efficient on graphs of girth at least 7 and on chordal bipartite graphs.

The presentation is based on joint works with following co-authors: Jean-Francois Couturier, Petr Golovach, Pim van 't Hof, Mamadou Kanté, Dieter Kratsch, and Yngve Villanger.

Optimising an imperfect tournament

Brett Stevens (Carleton University, Canada)

A computer science department holds an annual video game olympics with 64 participants playing 8 games. There are 8 rooms each with a fixed video game and there are 8 rounds. In each round 8 people will be in each room. Every person will play each game exactly once. We would like to find a schedule for all the players, rooms and rounds that is as balanced as possible, i.e. no pair of players plays together in the same room too frequently and as few pairs of people playing together are missed. It can be shown that some pairs must be missed and some pairs must repeat. We set up a combinatorial framework to quantify the repetition and missing pairs and try several different approaches to optimize the tournament. For various criteria we will find solutions constructed from lines in finite planes, ovals in finite geometries and finally a set of solutions that are related to Costas sonar and radar sequences and Almost Perfectly Non-linear functions.

The sum-product problem

Misha Rudnev (University of Bristol)

The original sum-product problem states, roughly, that for any finite set of integers A, the size of the set of all pairwise sums or products is always almost as large as the size of A, squared. 'Almost' refers to the fact that the set of pair-wise products of integers from 1 up to N has density about $1/\log\log N$.

The talk will discuss the state of the art of a few variants of this by and large wide-open question, asked over integers, reals, complex numbers, and finite fields.