Open University
Winter Combinatorics Meeting

Wednesday 22 January 2003

Timetable

10:45 - 10:50	Welcome and introduction John Mason, Sub-Dean (Research), Faculty of Mathematics and Computing
10:50 - 11:30	Martin Skoviera, Comenius University, Bratislava *Colouring cubic graphs by Steiner triple systems*
11:30 - 11:50	Tea/Coffee
11:50 - 12:30	Ian Anderson, University of Glasgow *Balancing training schedules for carryover effects*
12:30 - 14:00	Lunch
14:00 - 14:40	Stephanie Perkins, University of Glamorgan *Variable length codes that synchronize*
14:45 - 15:25	Gareth Jones, University of Southampton *Graphs, groups and surfaces*
15:25 - 15:45	Tea/Coffee
15:45 - 16:25	Douglas Woodall, University of Nottingham *Hall-type conditions: irreducibility and colourings*

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Colouring cubic graphs by Steiner triple systems

Martin Skoviera, Comenius University, Bratislava

Let $\mathcal{S} = (\mathcal{X}, \mathcal{B})$ be a *Steiner triple system*, a collection B of three-element subsets of a finite set X of points such that each pair of points is together present in exactly one triple. An $\mathcal{S}$-*colouring* of a cubic graph G is a colouring of the edges of G by points of $\mathcal{S}$ such that the colours of any three edges meeting at a vertex form a triple of $\mathcal{S}$. Note that the usual 3-edge colourings (commonly known as *Tait colourings*) are nothing but $\mathcal{I}$-colourings where $\mathcal{I}$ is the trivial Steiner triple system on three points. The study of edge-colourings of cubic graphs by Steiner triple systems was initiated by Archdeacon who asked the following general question: *Which cubic graphs can be coloured by which Steiner triple systems?*

In 2001, Fu identified two classes of cubic graphs which can be coloured by the unique Steiner triple system of order 7, the Fano plane. The first of these consists of all bridgeless cubic graphs of order at most 189, and the second comprises all such graphs of genus at most 24.

In the talk we will present a substantial improvement of these results. In particular, we will discuss the following two theorems:

Theorem A Let G be a bridgeless cubic graph and let $\mathcal{S}$ be a Steiner triple system of order greater than 3. Then G is $\mathcal{S}$-colourable.

Theorem B Let G be a cubic graph and let $\mathcal{S}$ be a projective Steiner triple system. Then G is $\mathcal{S}$-colourable if and only if G is bridgeless.

The above two results do not guarantee the existence of a single Steiner triple system which would colour every simple cubic graph. In fact, there are infinitely many Steiner triple systems which cannot colour any cubic graph with bridges (by Theorem B). Nevertheless, we are able to construct a Steiner triple system on 381 points and show that it can be used to colour every simple cubic graph. It is very likely, however, that there may exist smaller systems with a similar property.

The talk is based on joint results with Mike Grannell, Terry Griggs, Fred Holroyd, and Martin Knor.

Balancing training schedules for carryover effects

Ian Anderson, University of Glasgow

I will discuss some problems relating to the construction of training schedules for athletes, satisfying various balance requirements.

Variable length codes that synchronize

Stephanie Perkins, University of Glamorgan

Variable length codes are often used for data transmission. However, even a single bit error may cause the loss of some or all of the subsequent data, due to the decoder losing synchronization. Codes have been devised that enable the decoder to resynchronize quickly after an error has occurred and hence correctly decode the codewords which follow.
In this talk we will be particularly concerned with

- codes that contain a special codeword called a synchronizing codeword, which re-synchronizes the code whenever it is received correctly. Short synchronizing codewords are of special interest as they are transmitted frequently and hence synchronize the code more often. We will look at the structure of the synchronizing codewords, the existence of codes containing them and the robustness of these codes.
- codes that contain and extended synchronizing codeword which can be used with an appropriate scheme to prevent both error propagation and symbol shift errors.

Graphs, groups and surfaces

Gareth Jones, University of Southampton

My aim in this talk is to show how certain combinatorial structures, namely maps (embeddings of graphs in surfaces), are closely related to objects in other areas of mathematics, such as permutation groups, Riemann surfaces, algebraic number fields and Galois groups.

Hall-type conditions: irreducibility and colourings

Douglas Woodall, University of Nottingham

This talk is based on two joint papers with Sasha Kostochka.
A finite hypergraph (i.e., a finite family of finite sets $(A_i : i \in I)$) is *panchromatically t-colourable* if the vertices (elements) can be coloured with t colours in such a way that every colour is present on every edge (set). A *Hall-type condition* is a condition of the form

$$\left|\bigcup_{i \in J} A_i\right| \geq r|J| + \delta \quad \forall\, J \subseteq I.$$

A hypergraph is *irreducible* subject to a condition if it satisfies the condition but the removal of any element from any set destroys the condition.
In the first paper we investigated Hall-type sufficient conditions for a hypergraph to be panchromatically t-colourable (or t-choosable).
Historically this problem was first tackled by observing that, in certain cases, irreducible hypergraphs are *uniform* (i.e., have constant edge-size).
However, we found that this is far from true in general, and in the more recent paper we investigate how large an edge of an irreducible hypergraph can be.