Open University
Winter Combinatorics Meeting

Wednesday 20 January 2010

The talks will take place in Systems Seminar Room S0049, Venables Building, on the Open University campus in Milton Keynes.

Timetable

10:15 - 10:45	Tea/Coffee (in Mathematics Common Room)
10:50 - 10:55	Welcome and introduction (Robin Wilson)
11:00 - 11:40	Lowell Beineke, Purdue-Indiana University & Mathematical Institute, Oxford *Line graphs and beyond*
11:45 - 12:25	Maura Paterson, Royal Holloway, University of London *Multihop properties of distinct difference configurations*
12:30 - 13:55	Lunch
14:00 - 14:40	Leslie Ann Goldberg, University of Liverpool *Stable marriages and independent sets in bipartite graphs*
14:45 - 15.25	Jozef Skokan, London School of Economics *Ramsey goodness and bandwidth*
15:30 - 15:55	Tea/Coffee (in Mathematics Common Room)
16:00 - 16:40	Dan Archdeacon, University of Vermont *Superthrackles*

The meeting is financially supported by the British Combinatorial Committee.

Abstracts

Superthrackles

Dan Archdeacon, University of Vermont

A *thrackle* of a graph is a drawing in the plane so that every pair of non-adjacent edges cross exactly once. John Conway conjectured that if a connected graph is thracklable, then it has no more edges than vertices. A *superthrackle* is a drawing of a graph so that every pair of edges, adjacent or non-adjacent, cross exactly once. While Conway's original Thrackle Conjecture remains unsolved, we completely characterize those graphs that can be superthrackled. Is that all there is? No, we also introduce *superduperthrackles* and consider related problems on other surfaces.

This talk has lots of pretty pictures. It is joint work with Kirsten Stor.

Line graphs and beyond

Lowell Beineke, Purdue-Indiana University & Mathematical Institute, Oxford

In this talk, three variations on the definition and characterizations of line graphs will be explored:

- Super line graphs, based on sets of size greater than 1.
- The Krausz dimension of a graph, based on edge-decompositions with the maximum number of subgraphs containing a given vertex allowed to be greater than 2.
- Sets of excluded induced subgraphs for various families of graphs.

Stable marriages and independent sets in bipartite graphs

Leslie Ann Goldberg, University of Liverpool

The classical *stable marriage problem* is described as follows.

Given n men and n women, where each person has ranked all members of the opposite sex with a unique number between 1 and n in order of preference a *stable matching* is a bijection from the men to the women such that there are no two people of opposite sex who would both rather have each other than their current partners. Gale and Shapley showed in 1962 that a stable matching always exists and they gave an efficient algorithm for constructing one.

Some recent attention has focused on the problem of selecting a stable matching uniformly at random. Bhatnagar, Greenberg and Randall considered a certain natural Markov chain, and showed that it is slowly mixing, even when stable marriage instances are drawn from restricted geometric models.

It turns out that, even in these restricted models, the problem of selecting a stable matching uniformly at random is closely connected to the (still open) problem of sampling an independent set uniformly at random in a bipartite graph, which in turn is equivalent to many well-known sampling problems and is complete in a logically-defined complexity class.

This is joint work with Prasad Chebolu and Russell Martin.

Multihop properties of distinct difference configurations

Maura Paterson, Royal Holloway, University of London

A distinct difference configuration is a set of points in $\mathbb{Z}^2$ with the property that the vectors connecting any two of the points are all distinct. Motivated by an application of these structures in key predistribution for wireless sensor networks, we define the k-hop coverage of a distinct difference configuration to be the number of distinct vectors that can be expressed as the sum of k or fewer of the vectors that connect points of the configuration. We discuss bounds on the possible k-hop coverage of a distinct difference configuration, and consider the construction of configurations whose k-hop coverage is as great as possible.

Ramsey goodness and bandwidth

Jozef Skokan, London school of Economics

Given two graphs G and H, the Ramsey number $R(G, H)$ is the smallest N such that, however the edges of the complete graph K_N are coloured with red and blue, there exists either a red copy of G or a blue copy of H.

Adapting an earlier construction of Chvátal and Harary, Burr gave a simple general lower bound on the Ramsey number $R(G, H)$ for connected graphs G. Let $\sigma(H)$ be the smallest size of any colour class in any colouring of H by $\chi(H)$ colours. Then

$$R(G, H) \geq (\chi(H) - 1)(|G| - 1) + \sigma(H).$$

Here $|G|$ denotes the number of vertices of G and $\chi(H)$ is the chromatic number of H. For a given graph H, it is natural to ask which connected graphs G we have $R(G, H) = (\chi(H) - 1)(|G| - 1) + \sigma(H)$. We call such graphs H-*good*.

In this talk we will give an overview of some known results about Ramsey goodness, talk about open problems and offer some new results for cases when $G = H$ and when $|G|$ is much larger than $|H|$. In particular, we shall explore connections between Ramsey goodness and the bandwidth.

This is joint work with Peter Allen and Graham Brightwell.