Open University
Winter Combinatorics Meeting

Wednesday 24 January 2007

The talks will take place in room CMR11 on the Open University campus in Milton Keynes.

Timetable

10:15 - 10:45	Tea/Coffee (in M Block, Room Q229)
10:50 - 10:55	Welcome and introduction Uwe Grimm, Associate Dean (Research), Faculty of Mathematics and Computing
11:00 - 11:40	Bill Jackson, Queen Mary, University of London *Compatible circuit decompositions of 4-regular graphs*
11:45 - 12:25	Daniela Kühn, University of Birmingham *Generalized matching problems*
12:30 - 13:55	Lunch
14:00 - 14:40	Vassili Mavron, Prifysgol Cymru (University of Wales), Aberystwyth *Regular Hadamard matrices, codes and quasi-symmetric designs*
14:45 - 15.25	John Talbot, University College, University of London *G-intersecting families of sets*
15:30 - 15:55	Tea/Coffee (in M Block, Room Q229)
16:00 - 16:40	Imre Leader, Trinity College, University of Cambridge *Intersecting families of permutations*

The meeting is financially supported by the British Combinatorial Committee.

Abstracts

Compatible circuit decompositions of 4-regular graphs

Bill Jackson, Queen Mary, University of London

The circuit double cover conjecture asserts that the edges of a bridgeless graph G can be covered with circuits in such a way that each edge of G appears in exactly two circuits. An obvious, but fallacious, proof would be to 'double' each edge of G and then choose a circuit decomposition of the resulting Eulerian graph $2G$. We can try to repair the hole in this 'proof' by putting restrictions on which edges can be consecutive in the circuit decomposition of $2G$. This gives rise to the notion of a compatible circuit decomposition. I will present a conjectured characterization of when a 4-regular graph has a compatible circuit decomposition and show that it is equivalent to an old conjecture that every sufficiently connected 4-regular graph has a compatible circuit decomposition. This is joint work with Herbert Fleischner and François Genest.

Generalized matching problems

Daniela Kühn, University of Birmingham

A classical result of Tutte characterizes all those graphs which have a perfect matching. In my talk I will consider the following more general question: Given two graphs H and G, when does there exist a collection of pairwise disjoint copies of H in G which covers all the vertices of G? Such a collection is called a perfect H-packing. (So a perfect matching in G corresponds to the case when H is an edge.) It is unlikely that an analogue of Tutte's theorem exists for this problem, so I will consider conditions on the minimum degree of G which ensure the existence of a perfect H-packing in G. This is joint work with Deryk Osthus.

Regular Hadamard matrices, codes and quasi-symmetric designs

Vassili Mavron, Prifysgol Cymru (University of Wales), Aberystwyth

The connection between $4n \times 4n$ Hadamard matrices and symmetric $2-(4n-1, 2n-1, n-1)$ designs and affine $3-(4n, 2n, n-1)$ designs is well-known.

A regular $4n \times 4n$ Hadamard matrix is one whose row sums are all equal; in which case $n = u^2$ is necessarily a square. Such a Hadamard matrix has an additional symmetric 2-design associated with it - one with parameters

$$2-(4u^2, 2u^2-u, u^2-u)$$

Conversely, from a design with these parameters one may construct a regular $4u^2 \times 4u^2$ Hadamard matrix.

There does not appear to be an additional affine design associated with regular Hadamard matrices. McGuire (1997) established the equivalence of the existence of self-complementary codes meeting the Grey-Rankin bound and the existence of certain quasi-symmetric designs. In a later paper, Bracken, McGuire and Ward (2006) proved the existence of some new quasi-symmetric designs by constructing self-complementary codes meeting the Grey-Rankin bound, using two constructions with u or $u+1$ latin squares of order $2u$, and a Hadamard $2u \times 2u$ matrix (not necessarily regular).

The quasi-symmetric designs in the Bracken et al. paper appear to have the parameters of the residual and a derived designs of a symmetric $2-(4u^2, 2u^2-u, u^2-u)$ design. It was not known whether or not they can be embedded in some such a design. (Affine designs are quasi-symmetric and quasi-residual.)

Joint work with T P McDonough has shown that the Bracken et al. results can be proved by direct geometric design theory construction, without using codes or the Grey-Rankin bound. Further, we can show that they the quasi-symmetric designs are embeddable in certain cases .

It is interesting to note that the existence of the latin squares alone implies the existence of a $2-(4u^2, 2u^2-u, u^2-u)$ design, which is rather special in that it is constructed from a strongly-regular graph and has a null-polarity. However, there does not appear to be any connection between this design and the two quasi-symmetric designs; this is perhaps not surprising since the Hadamard $2u \times 2u$ matrix is not used in its construction.

G-intersecting families of sets

John Talbot, University College, University of London

The classical Erdős–Ko–Rado theorem tells us how large an intersecting family of k-sets from a ground set of size n can be. I will discuss a related problem in which we allow the sets in our family to be disjoint but insist that they are "close" in some sense.

In order to make this precise Bohman, Frieze, Ruszinkó and Thoma formulated the following natural definition: if $G = (V, E)$ is a graph then a family of subsets of V is *G-intersecting* if any two sets from the family either meet or contain adjacent vertices.

I will consider the question of how large a G-intersecting family of k-sets can be. Surprisingly it turns out that both parts of the Erdős–Ko–Rado theorem can have non-trivial analogues in this new setting.

This is joint work with Robert Johnson (QMUL).

Intersecting families of permutations

Imre Leader, Trinity College, University of Cambridge

A family of permutations on an n-point set is said to be *intersecting* if, for any two of them, some element is sent to the same place by both. How large can such a family be? Similarly, a family of partial permutations (meaning injections defined only at some r points of the n-set, for some fixed r) is *intersecting* if, for any two of them, there is some element where they are both defined and that is sent to the same place by both. Again, how large can such a family be?

The first of these is an (old and) easy result, while the second is rather harder. We will discuss these questions, and go on to mention some of the fascinating open problems about intersecting families of permutations.

This is joint work with C.Y.Ku.