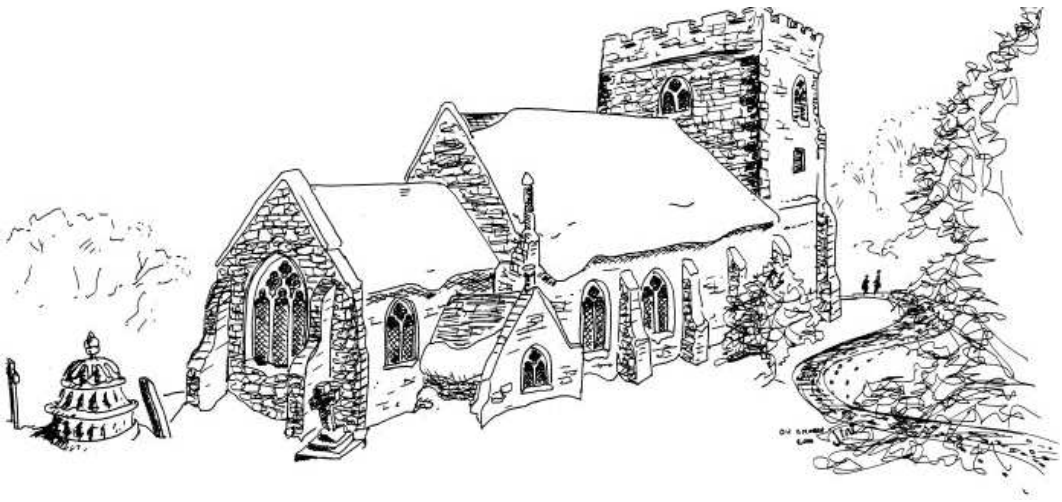
The Open University

LONDON MATHEMATICAL SOCIETY

British Combinatorial Committee

# Open University Winter Combinatorics Meeting

## Wednesday 30 January 2013



# Open University
# Winter Combinatorics Meeting

## Wednesday 30 January 2013

**The talks will take place in Christodoulou Meeting Room 11, on the Open University campus in Milton Keynes.**

### Timetable

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| 10:15 - 11:00 | Tea/Coffee (in the Mathematics and Statistics Common Room) |
| 11:00 - 11:40 | David Conlon (Oxford University)<br>*Extremal results in sparse pseudorandom graphs* |
| 11:45 - 12:25 | Steven Noble (Brunel University)<br>*The Merino-Welsh conjecture: an inequality for Tutte polynomials* |
| 12:30 - 13:55 | Lunch |
| 14:00 - 14:40 | Iain Moffat (Royal Holloway)<br>*Partial duals of embedded graphs* |
| 14:45 - 15.25 | David Evans (University of East Anglia)<br>*Matroids in model theory* |
| 15:30 - 15:55 | Tea/Coffee (in the Mathematics and Statistics Common Room) |
| 16:00 - 16:40 | Mireille Bousquet-Mélou (Université Bordeaux 1, France)<br>*Asymptotic properties of some minor-closed classes of graphs* |

The meeting is financially supported by the London Mathematical Society and the British Combinatorial Committee.

Drawing of St Michael's church, on the OU campus, by Jini Williams.

# Abstracts

### Extremal results in sparse pseudorandom graphs

David Conlon (Oxford University, UK)

Szemerdi's regularity lemma is a fundamental tool in extremal combinatorics. However, the original version is only helpful in studying dense graphs. In the 1990s, Kohayakawa and Rödl proved an analogue of Szemerédi's regularity lemma for sparse graphs as part of a general program toward extending extremal results to sparse graphs. Many of the key applications of Szemerédi's regularity lemma use an associated counting lemma. In order to prove extensions of these results which also apply to sparse graphs, it remained a well-known open problem to prove a counting lemma in sparse graphs.

In this talk we discuss a new counting lemma, proved following the functional approach of Gowers, which complements the sparse regularity lemma of Kohayakawa and Rödl, allowing us to count small graphs in regular subgraphs of a sufficiently pseudorandom graph. We use this to prove sparse extensions of several well-known combinatorial theorems, including the removal lemmas for graphs and groups, the Erdős-Stone-Simonovits theorem and Ramsey's theorem. These results extend and improve upon a substantial body of previous work.

### The Merino-Welsh conjecture: an inequality for Tutte polynomials

Steven Noble (Brunel University, London, UK)

While investigating convexity properties of the Tutte polynomial, Criel Merino and Dominic Welsh conjectured that in any 2-connected loopless, bridgeless graph, the larger of the number of acyclic orientations and the number of totally cyclic orientations is at least the number of spanning trees of the graph. Each of these invariants is an evaluation of the Tutte polynomial. We will discuss the background to this conjecture and explain why various 'obvious' approaches do not work. We show that a stronger version of it holds for series-parallel networks. This is joint work with many co-authors including, for the most recent result, Gordon Royle.

**Partial duals of embedded graphs**

Iain Moffat (Royal Holloway, London, UK,
moving from University of South Alabama, USA)

The geometric dual $G^*$ of an embedded graph G is a fundamental construction in graph theory, and is one that appears in many areas of mathematics. Recently, S. Chmutov generalised geometric duality by introducing the idea of a partial dual. Roughly speaking, a partial dual of an embedded graph is obtained by forming the geometric dual with respect to only some of its edges. In this talk I will describe partial duality, and give an overview of some of its properties and applications. I will pay particular attention to partial duals of low genus graphs (while geometric duality preserves genus, partial duality does not), and their applications to knot theory. Throughout the talk, I will emphasise how the subject arises as an attempt to lift classical results and constructions for plane graphs off of the plane.

**Matroids in model theory**

David Evans (University of East Anglia, Norwich, UK)

I will explain how questions about matroids arise naturally in model theory and discuss how certain highly influential model-theoretic constructions of combinatorial structures due to Hrushovski in the late 1980's are related to the strict gammoids (or cotransversal matroids) studied by matroid theorists in the early 1970's.

**Asymptotic properties of some minor-closed classes of graphs**

Mireille Bousquet-Mélou (Université Bordeaux 1, France)
(joint work with Kerstin Weller, Oxford)

Let $A$ be a minor-closed class of labelled graphs, and let $G_n$ be a random graph sampled uniformly from the set of $n$-vertex graphs of $A$. When $n$ is large, what is the probability that $G_n$ is connected? How many components does it have? How large is its biggest component? Thanks to the work of McDiarmid and his collaborators, these questions are now solved when all excluded minors are 2-connected.

Using exact enumeration, we study a collection of classes $A$ excluding non-2-connected minors, and show that their asymptotic behaviour may be rather different from the 2-connected case. This behaviour largely depends on the nature of dominant singularity of the generating function $C(z)$ that counts connected graphs of $A$. We classify our examples accordingly, thus providing a first step towards a complete classification of minor-closed classes of graphs.