Open University Winter Combinatorics Meeting

Wednesday 23 January 2002

Schedule

10:45 - 10:50	Welcome and introduction Professor Brenda Gourley, Vice-Chancellor
10:50 - 11:30	Darryn Bryant, University of Queensland, Australia Completing latin rectangles to symmetric latin squares
11:30 - 11:50	Morning Tea/Coffee
11:50 - 12:30	Stephanie Perkins, University of Glamorgan Variable length codes that synchronize
12:30 - 14:00	Lunch
14:00 - 14:40	Kenny Paterson, Royal Holloway, University of London Universal cycles: de Bruijn and beyond
14:45 - 15:25	Ray Hill, University of Salford Optimal linear codes and finite geometries
15:25 - 15:45	Afternoon Tea/Coffee
15:45 - 16:25	Norman Biggs, London School of Economics Group representation and chromatic polynomials

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Completing latin rectangles to symmetric latin squares

Darryn Bryant, University of Queensland, Brisbane, Australia

An n by n (partial) latin square is an n by n array of cells, each (either empty or) containing a symbol chosen from an n-set such that each symbol occurs exactly (at most) once in each row and exactly (at most) once in each column. A partial latin square is said to be completable to a latin square if its empty cells can be filled so that a latin square results. An m by n ($m \leq n$) latin rectangle is a partial latin square in which the first m rows are filled and the remaining rows are empty. A latin square is symmetric if for i and j in the range 1 to n, cells (i, j) and (j, i) contain the same symbol.

It follows from Hall's theorem on systems of distinct representatives that any m by n latin rectangle can be completed to an n by n latin square. However, deciding which m by n latin rectangles are completable to symmetric latin squares is a more difficult problem in general. Necessary and sufficient conditions for a 2 by n latin rectangle to be completable to an n by n symmetric latin square will be presented together with an outline of the proof and discussion of related results.

Variable length codes that synchronize

Stephanie Perkins, University of Glamorgan

Variable length codes are often used for data transmission. However, even a single bit error may cause the loss of some or all of the subsequent data, due to the decoder losing synchronization. Codes have been devised that enable the decoder to resynchronize quickly after an error has occurred and hence correctly decode the codewords which follow.

In this talk we will be particularly concerned with

- codes that contain a special codeword called a synchronizing codeword, which re-synchronizes the code whenever it is received correctly. Short synchronizing codewords are of special interest as they are transmitted frequently and hence synchronize the code more often. We will look at the structure of the synchronizing codewords, the existence of codes containing them and the robustness of these codes.

- codes that contain and extended synchronizing codeword which can be used with an appropriate scheme to prevent both error propagation and symbol shift errors.

Universal cycles: de Bruijn and beyond

Kenny Paterson, Royal Holloway, University of London

The period 16 cyclic sequence s of 0's and 1's:

$$s = 0000111101100101\ldots$$

has the following nice property: Take a piece of card, cut a window in it that is exactly 4 symbols long and place the card over the sequence. Then, in sliding 'discretely' along one period of s, we will see every possible 4-tuple of symbols through the window exactly once each. Sequence s, called a span 4 de Bruijn cycle, has the property that every element from the set of binary 4-tuples appears in s, and s has minimum period. De Bruijn cycles were 'discovered' by de Bruijn in 1946, but actually date back at least as far as Flye-Sainte Marie in 1894.

In this talk we'll consider Universal Cycles (Hurlbert, 1990, Chung, Diaconis and Graham, 1992): these are periodic sequences in which the n-tuples of consecutive positions represent in a natural way all the elements of some set X exactly once each in a period. So a span n de Bruijn sequence is a Universal Cycle for the set X of binary n-tuples. Other sets X that have been considered so far include k-permutations of an n-set, k-subsets of an n-set, partitions of an n-set, and so on. As just one example to whet the appetite, it is easy to check that:

$$123423142132432143124134\ldots$$

is a Universal Cycle for 3-permutations of a 4-set. Knuth has computed that there are 384 such sequences.

Given a set X, the main questions that we study are:

1. Does a Universal Cycle for X exist?
2. If so, how many Universal Cycles are there?
3. Is there an efficient algorithm for generating some (or even all) Universal Cycles for X?
4. Given a description of a Universal Cycle for X and an element x in X, is there an efficient algorithm for computing the position in the Cycle of the n-tuple representing x?

In this talk, we'll sketch some of what's known for each of these questions, focussing mostly on the case where X is the set of n-tuples or the set of k-permutations of an n-set.

Optimal linear codes and finite geometries

Ray Hill, University of Salford

A linear $[n, k, d]$ code over $GF(q)$ is said to be optimal if its length n is smallest possible for given values of the dimension k and minimum distance d. It will be shown that the optimal linear codes problem, namely that of finding this smallest length and classifying the codes which achieve it, is most naturally considered as a problem in finite geometry. This will be illustrated by constructions of many optimal codes, old and new, based on various combinatorial structures such as conics, quadrics, arcs, caps and blocking sets.

Group Representations and Chromatic Polynomials

Norman Biggs, London School of Economics

In Statistical Physics, critical behaviour (such as change of state) is explained in terms of the singularities of the partition function of a suitable model. The chromatic function of a graph is a special case. This function has two very obvious properties. First, if the number k of colours is larger than the number of vertices of the graph, not all the colours can be used. This means that the function is polynomial in k. Secondly, the names of the colours are unimportant, and so the symmetric group $\mathrm{Sym}(k)$ can be made to act on the set of colours.

In the talk I shall describe how the second of these properties can be used to obtain formulae for the chromatic polynomials of 'bracelets'. These are graphs that mimic the lattice-like structures studied in physics. The resulting formulae are well-adapted to studying the behaviour of the zeros of the polynomials as the number of vertices tends to infinity.