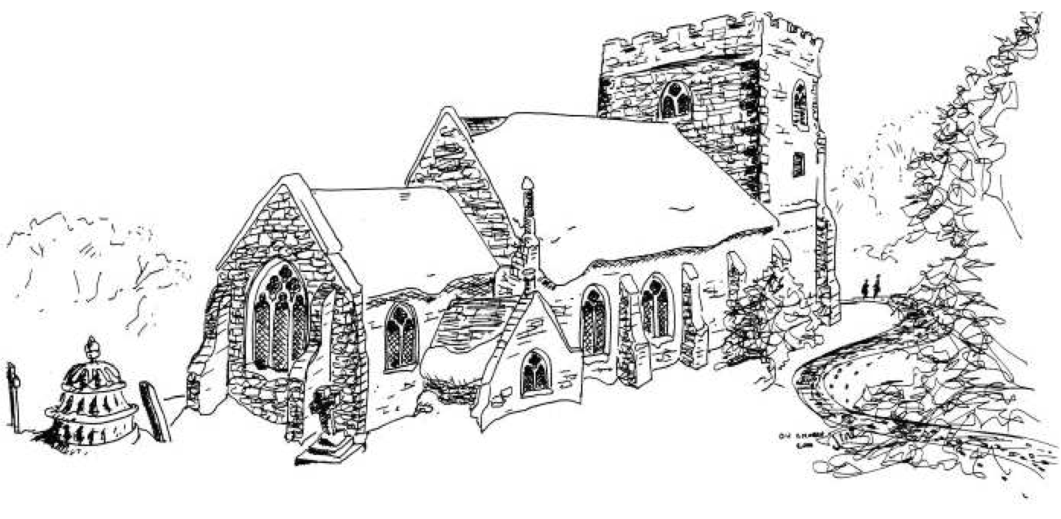
The Open University

British Combinatorial Committee

Open University
Winter Combinatorics Meeting

Wednesday 4 March 2015



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The talks will take place in Christodoulou Meeting Room 15, on the Open University campus in Milton Keynes.

Timetable

10:15 - 11:00	Tea/Coffee (in the foyer of Room 15)
11:00 - 11:40	Klara Stokes (Linköping University, Sweden) *Linear patterns of ideals of numerical semigroups (with applications to configurations)*
11:45 - 12:25	Robert Jajcay (Comenius University, Slovakia and University Primorska, Slovenia) *Symmetric cages*
12:30 - 13:55	Lunch
14:00 - 14:40	Maurice Pouzet (Université Claude Bernard Lyon 1 and University of Calgary, Canada) *Decompositions of relational structures and enumeration*
14:45 - 15.25	Christina Goldschmidt (University of Oxford) *The scaling limit of the minimum spanning tree of the complete graph*
15:30 - 15:55	Tea/Coffee (in the foyer of Room 15)
16:00 - 16:40	Vadim Lozin (Université Bordeaux 1, France) *Deciding the Bell number for hereditary graph properties*

The meeting is financially supported by the British Combinatorial Committee.

Drawing of St Michael's church, on the OU campus, by Jini Williams.

Abstracts

Linear patterns of ideals of numerical semigroups (with applications to configurations)
Klara Stokes

A numerical semigroup is a subset of the natural numbers which contains 0, is closed under addition and has finite complement. Numerical semigroups have applications in algebraic geometry as Weierstrass semigroups, in commutative algebra as the value semigroups of one-dimensional Noetherian local analytically irreducible domains and in combinatorics as the semigroups of configurable parameters of (r, k)-configurations. An ideal I of a numerial semigroup S is a set $I \subseteq S$ satisfying $I + S \subseteq I$. A linear pattern admitted by an ideal of a numerical semigroup S is a multivariate polynomial function $p(X_1, ..., X_n)$ which returns an element $p(s_1, ..., s_n) \in S$ when evaluated on any non-increasing sequence $(s_1, ..., s_n)$ of elements in S. I will give results regarding the image of an ideal under a pattern and on the structure of linear patterns admitted by a numerical semigroups. I will also show how this applies to the existence question of combinatorial configurations.

Symmetric cages
Robert Jajcay

A (k, g)-cage is an optimal graph in which every vertex is of the same degree k and every cycle is of length at least g. The optimality of a (k, g)-cage lies in its being the smallest among all graphs with these properties. This combination of requirements makes for extremely intriguing objects most often exhibiting very high levels of symmetry. Thus focusing on classes of graphs with rich automorphism groups, specifically on vertex-transitive or Cayley graphs, serves the double purpose of searching through potentially promising classes and opening the possibility for using algebra and group theory. We present several examples of such approach to the problem of finding cages with the aim of introducing the audience to the beauty and excitement of the area as well as to the power of the use of algebra in combinatorics.

Decompositions of relational structures and enumeration
Maurice Pouzet

I will report on some joint work with N. Thiéry and with D. Oudrar on the notion of monomorphic decomposition of a relational structure. This notion allows to explain, in part, the jump phenomena of the enumerative functions of hereditary classes of finite ordered structures, e.g. their growth rate is either polynomial or exponential (Kaiser and Klazar (2003) and Balogh, Bollobás and Morris (2006)). I will briefly present a notion of cellular decomposition of a relational structure directly inspired from a notion invented by Schmerl (1990). It could explain the jump phenomenon also observed in the case of structures which are not necessarily ordered and play a role in a longstanding conjecture of Cameron on his age algebra.

The scaling limit of the minimum spanning tree of the complete graph
Christina Goldschmidt

Consider the complete graph on n vertices with independent and identically distributed edge-weights (say uniform on $[0, 1]$). The minimum spanning tree (MST) is simply the spanning subtree of smallest weight. It is straightforward to construct the MST using one of several greedy algorithms. Kruskal's algorithm builds the tree edge by edge starting from the globally lowest-weight edge and then adding other edges one by one in increasing order of weight, as long as they do not create any cycles. At each step of this process, the algorithm has generated a forest, which becomes connected on the final step. In this talk, I will explain how it is possible to exploit a connection between the forest generated by Kruskal's algorithm and the Erds-Rnyi random graph in order to prove that M_n, the MST of the complete graph, possesses a scaling limit as n tends to infinity. In particular, if we think of M_n as a metric space (using the graph distance), rescale edge-lengths by $n^{-1/3}$, and endow the vertices with the uniform measure, then M_n converges in distribution a certain random real tree.

This is joint work with Louigi Addario-Berry (McGill), Nicolas Broutin (INRIA Paris-Rocquencourt) and Grégory Miermont (ENS Lyon).

Deciding the Bell number for hereditary graph properties

Vadim Lozin

The paper [J. Balogh, B. Bollobas, D. Weinreich, A jump to the Bell number for hereditary graph properties, *J. Combin. Theory Ser. B* 95 (2005) 29-48] identifies a jump in the speed of hereditary graph properties to the Bell number B_n and provides a partial characterisation of the family of minimal classes whose speed is at least B_n. In this talk, we give a complete characterisation of this family. Since this family is infinite, the decidability of the problem of determining if the speed of a hereditary property is above or below the Bell number is questionable. We answer this question positively by showing that there exists an algorithm which, given a finite set F of graphs, decides whether the speed of the class of graphs containing no induced subgraphs from the set F is above or below the Bell number. For properties defined by infinitely many minimal forbidden induced subgraphs, the speed is known to be above the Bell number.

Joint work with Aistis Atminas, Andrew Collins and Jan Foniok.