Open University
Winter Combinatorics Meeting

Wednesday 25 January 2006

Timetable

10:50 - 10:55	Welcome and introduction Professor Brigid Heywood, Pro-Vice-Chancellor (Research and Staff)
11:00 - 11:40	Martin Knor, Slovak University of Technology, Bratislava *Cycles in iterated line graphs*
11:45 - 12:25	Rosemary Bailey, Queen Mary, University of London *Association schemes and their products*
12:30 - 13:55	Lunch
14:00 - 14:40	Nigel Martin, University of Durham *Factorisations of complete bipartite graphs*
14:45 - 15.25	Simon Blackburn, Royal Holloway, University of London *Sets of permutations that generate the symmetric group pairwise*
15:30 - 15:55	Tea/Coffee (in M Block, Room Q229)
16:00 - 16:40	Diane Donovan, University of Queensland *From Latin squares to Steiner triple systems, via trades, uniquely completable sets and defining sets*

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Cycles in iterated line graphs

Martin Knor, Slovak University of Technology, Bratislava

A graph G is k-ordered if for every ordered sequence of k vertices, there is a cycle in G that encounters the vertices of the sequence in the given order. We prove that if G is a connected graph distinct from a path, then there is a number t_G such that for every $t \geq t_G$ the t-iterated line graph of G, $L^t(G)$, is $(\delta(L^t(G))+1)$-ordered. Since there is no graph H which is $(\delta(H)+2)$-ordered, the result is best possible.

Association schemes and their products

Rosemary Bailey, Queen Mary, University of London

An association scheme of rank r is a partition of the ordered pairs from a finite set into r parts; some conditions have to be satisfied which ensure that calculations about the scheme can be done in an r-dimensional commutative real algebra.

There are two classical ways of combining two assocation schemes on sets of size n and m to make one on a set of size mn: the direct product and the wreath product. I shall generalize these in two ways. One way makes a product of several association schemes such that the relationship between any two is similar to that in either a direct or wreath product. The other stays with two schemes but gives a family of products, of which the direct product and the wreath product are extreme cases.

Factorisations of complete bipartite graphs

Nigel Martin, University of Durham

Let G be a simple bipartite graph. A *G-factor* of the complete bipartite graph $K_{m,n}$ is a spanning subgraph that can be decomposed into vertex-disjoint copies of G. A *G-factorisation* is a decomposition of $K_{m,n}$ into edge-disjoint G-factors.

The question is then to determine necessary and sufficient conditions for such factorisations to exist for a given graph G.

Simple necessary conditions can usually be determined from arithmetic calculations based on numbers of vertices and edges. So are these sufficient?

We shall look at the current state of work for the cases where $G = P_k$, where P_k is a path of k edges, and $G = K_{p,q}$. The former case is easy when k is odd, but apart from $k = 2$ seems very difficult.

There has been much more progress in the case where G is itself complete bipartite, with the arithmetic conditions being shown to be sufficient whenever $m = n$ (the balanced case), as well as for a wide range of non-balanced situations.

In the non-balanced situations the copies of $G = K_{p,q}$ in any G-factor can be oriented in two ways depending whether the p-set of $K_{p,q}$ lies in the m-set or the n-set of $K_{m,n}$. The ratio of the two types is called the *balance ratio*. Suppose this is $x : y$ [where $\gcd(x, y) = 1$]. It turns out that the quantity $\gcd(p - q, x + y)$ has a significant role. Indeed the evidence suggests that the arithmetic conditions will be sufficient if $\min\{x, y\} >> \gcd(p - q, x + y)$. The techniques used are likely to fail when $x,\ y$ both have small values relative to $\gcd(p - q, x + y)$.

Sets of permutations that generate the symmetric group pairwise

Simon Blackburn, Royal Holloway, University of London

A subset X of a group G *generates G pairwise* if for all $g_1, g_2 \in X$ with $g_1 \neq g_2$ we have that g_1 and g_2 generate G. (Of course, this notion is only interesting when G is 2-generated!) We write $\mu(G)$ for the maximum size of a set X that generates G pairwise.

In this talk, we will motivate the study of $\mu(G)$, and sketch a proof of the result that $\mu(S_n) = 2^{n-1}$ when n is a sufficiently large odd integer. (Here S_n is the symmetric group on n letters.)

From Latin squares to Steiner triple systems, via trades, uniquely completable sets and defining sets

Diane Donovan, University of Queensland

This talk will present constructions for uniquely completable sets in Latin squares and the importance of Latin trades to this process. From here we will draw the connection with trades in Steiner triple systems and show how the constructions can be adapted to prove the of existence of defining sets in Steiner triple systems. Known results will be presented and future directions will be highlighted.